linear etch pattern might have been formed along with edges of imperfect layers deposited during growth. The calcite crystals were of rhombohedral habit. The growth layers make an angle of about 75° with the cleavage face. The displacement observed in Fig. 9 across the cleavage step and the height of the measured step are such that the angle made by the imperfect layers with the cleavage face is about 73°.

The question arises as to how far within the body of the crystal does such a pattern extend. We have obtained an answer to this question by using the method adopted by Patel & Tolansky (1957) for diamond. A small block of calcite was cleaved out and etched on all faces. A remarkable correlation appears in the patterns of the faces. These etch patterns are shown in Fig. 10 (×22) which is an exploded view, showing relation between the faces. Distinctive regions exhibiting different degrees of attack are seen. In one region the pits are widely distributed and in the other linear arrangements of densely populated pits are formed. These linear arrangements reveal the growth stratigraphy of the crystal.

It is clear that the stratigraphical pattern goes right through the body of crystal. Indeed, the etch has revealed the true history of growth of the calcite. The crystal sheets growing layerwise might have grown under different conditions. Such sheets of different thicknesses, maintain their individuality through the whole crystal block and this accounts for the similar etch pattern appearing on the four cleavage faces as shown in Fig. 10.

The implication seems to be that growth conditions (temperature pressure, impurities etc.), whilst effectively constant for each region differ markedly for successive regions.

It might well be that the growth is controlled by two separate rates:

(1) Sheets with isolated pits might have grown slowly.
(2) Sheets with densely populated pits have probably been deposited fairly rapidly.

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Plane Groups on Polyhedra

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Simple mosaics are given for the plane groups and the conditions for fitting these on the surfaces of polyhedra are discussed. All the basic polyhedra are considered, and a list is drawn up of the various possibilities. Some application to the structure of viruses is suggested.

Introduction

A mosaic is a two-dimensional array of congruent shapes which completely covers the plane without overlapping. For this array to belong to one of the 17 plane groups, it must be invariant to certain symmetry transformations. If the basic shape has no symmetry, the shapes must be in the general positions of the plane group.

The simplest possible such shape will give the simplest possible mosaic which fully describes the plane group. As any antisymmetry or colour-symmetry plane group is isomorphic with one of the plane groups, the mosaic for this plane group will be the most appropriate for adaptation for all the isomorphic groups.

Belov *et al.* (1956, 1957, 1958) have given mosaics for antisymmetry and colour-symmetry plane groups, but these do not all agree with the principles laid down above. Thus Belov & Belova (1957) use a mosaic isomorphic with Fig. 1 to describe the colour-symmetry groups numbered I*a* and III*a* in their paper. Fig. 1 contains numerous planes of symmetry, which do not become planes of true or colour symmetry in the diagrams of Belov & Belova. Moreover Fig. 1 is not the most appropriate mosaic to use to describe

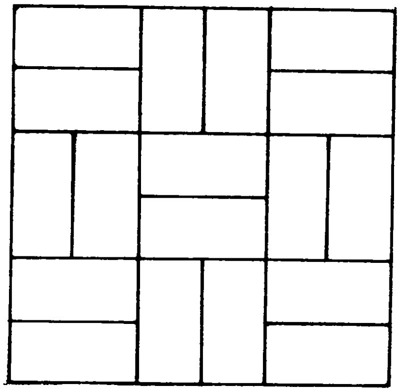

Fig. 1. Mosaic isomorphic with I(a) and III(a) of Belov & Belova (1957).

the symmetry group to which it belongs. A better mosaic is Fig. 2 No. 12.

Mosaics for the plane groups

Fig. 2 gives a suggestion for the simplest possible mosaics for the 17 groups.

Diagrams for Nos. 6, 11 and 14 cannot be obtained with a single shape as the basic shape unless the

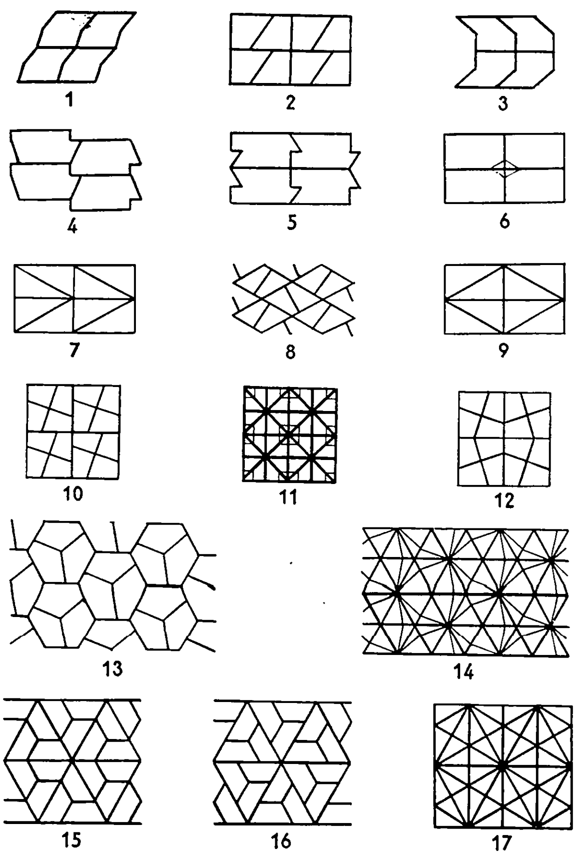

Fig. 2. Mosaics for the 17 plane groups.

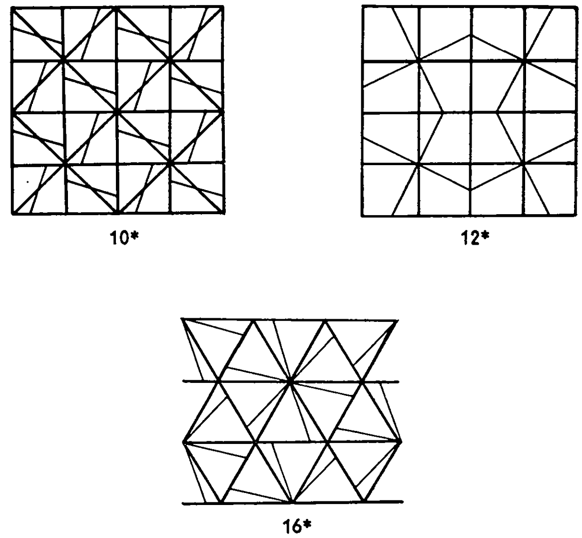

Fig. 3. Alternative mosaics for plane groups Nos. 10, 12 and 16.

symmetry of this basic shape is removed. This is done in the diagrams by drawing in a fine line. Alternative mosaics are given for plane groups 10, 12 and 16 in Fig. 3. The symmetry considerations are here satisfied where again the finer line removes any symmetry not belonging to the plane group.

Plane groups on polyhedra

In the plane mosaic each basic shape is surrounded by its nearest neighbours in an exactly similar way. It is the purpose of this paper to find the possible convex polyhedra that can be obtained by applying only the restriction that around each shape the nearest neighbours must be the same as in the plane group. Thus in the plane group 16 (Fig. 2), where we have six areas around one point, we can reduce the number to 5 and similarly about 11 other points and obtain the icosahedron Fig. 4 No. (i)(a). Clearly in

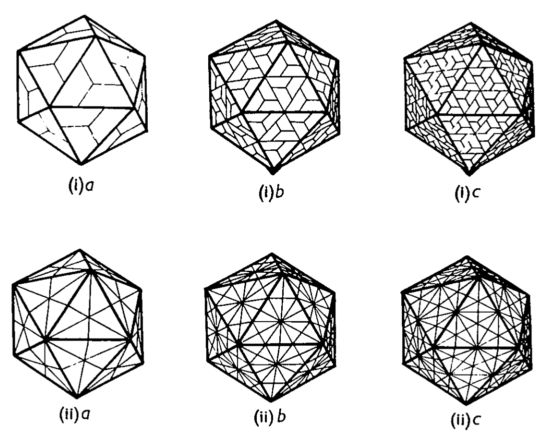

Fig. 4. Plane groups Nos. 16 and 17 on icosahedra.

this case the nearest neighbour environment is the same as in the plane group, but this would not be so for No. 15 Fig. 2. Thus this plane group does not fit on to an icosahedron. Similar consideration of the plane group No. 17 where twelve shapes surround some points leads to the icosahedron Fig. 4 No. (ii)(*a*).

The other icosahedra in Fig. 4 are produced from the plane groups Nos. 16 and 17 by fitting more shapes on to each face. This process can be continued indefinitely, but the number of shapes on the icosahedron is always an integral multiple of 60.

There are only eight convex polyhedra whose faces are equilateral triangles. Figs. 4, 5 and 6 show some possible arrangements of the plane groups on these polyhedra. The polyhedra which can be formed by combining tetrahedra and octahedra are not considered separately, as the condition that a plane group can fit on both is sufficient for it to fit on any combination of these polyhedra.

It is evident that for a plane mosaic to fit on to a polyhedron this mosaic must have a symmetry axis of higher than second order. Thus we only have to consider the trigonal, hexagonal and tetragonal plane groups. The tetragonal groups Nos. 10 and 11 are able to fit over the surface of a cube, as shown in Fig. 7, and can therefore fit over any rectangular parallelepiped with integral length edges. The alternative mosaic for the plane group No. 10 given in Fig. 3 is used for Fig. 7 Nos. (ii)(*a*), (*b*), (*c*), as this allows superpositions with a minimum of 12 shapes, whereas using No. 10 (Fig. 2) the minimum number is 24. This latter possibility is in fact equivalent to Fig. 7 No. (ii)(*b*).

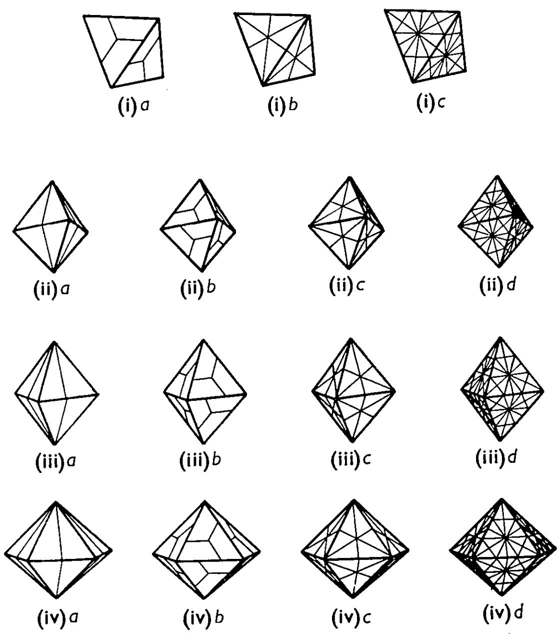

Fig. 5. Plane groups Nos. 16 and 17 on (i) tetrahedra, (ii) trigonal dipyramids, (iii) octahedra and (iv) pentagonal dipyramids.

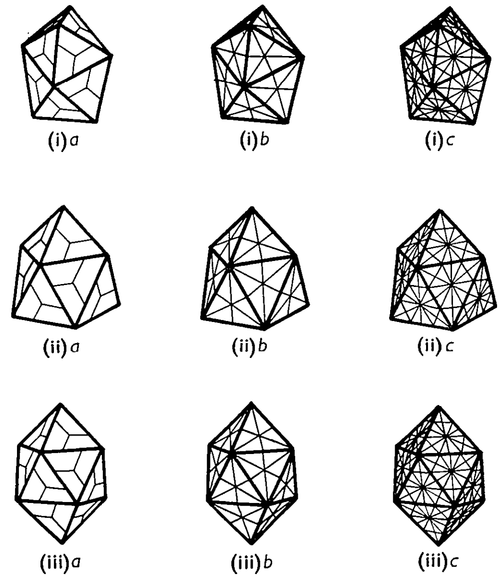

Fig. 6. Plane groups Nos. 16 and 17 on (i) 12-hedron, (ii) 14-hedron and (iii) 16-hedron.

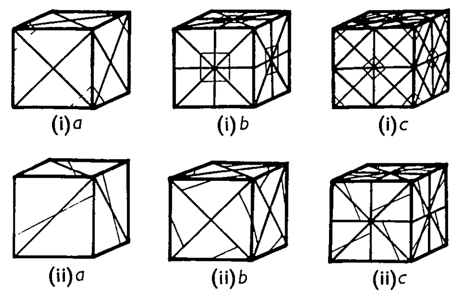

Fig. 7. Plane groups (i) 11, (ii) 10* (Fig. 3) on cubes.

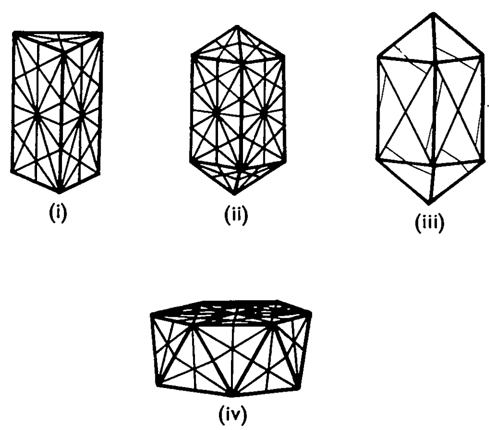

Fig. 8. Plane group (i) 17 on trigonal prism, (ii) 17 on tetragonal pyramidal prism, (iii) 10* (Fig. 3) on trigonal pyramidal prism, (iv) 17 on hexagonal antiprism.

Fig. 8 shows the superposition of mosaics on trigonal, tetragonal and hexagonal prisms and antiprisms, with or without pyramidal ends.

The polyhedra considered here do not exhaust the possibilities, as the class of compound polyhedra has been excluded. Table 1 gives a summary of the results.

The enumeration of the above possibilities was suggested by the symmetry of a Chinese hat, of which a photograph is given in Fig. 9. Three sets of parallel strands are woven at 60° to each other, and this produces a pattern belonging to the plane group No. 17. At one point the number of strands is restricted to five, and this point, the apex of the hat, is therefore a 5-fold axis. This is the first stage in the construction of an icosahedron of the type of (ii), Fig. 4.

The plane groups on polyhedra might have application to the structure of viruses. It has been shown

Table 1. *A summary of the results, giving the number of plane group units which fit on a polyhedron*

Polyhedron type	Point group	Plane group	Number of units		Point group (if new)	Figure
Prisms						
Orthorhombic	mmm	10*	$2a(lm+mn+nl)$	$a=2$ or 4	222	—
		11		$a=4$ or 8	—	—
		12*		$a=4$	—	—
Tetragonal	$4/mmm$	10*	$2a(2mn+n^2)$	$a=$ 2, m odd	2	—
				2, m even	222	—
				4	422	—
		11		$a=$ 4, m odd	$\bar{4}2m$	—
				4, m even	mmm	—
				8	$4/mmm$	—
		12*		$a=$ 4, m odd	mmm	—
				4, m even	$\bar{4}2m$	—
Cube	$m3m$	10*	$12n^2$, n odd		23	7(ii)(a)
			$12(2n)^2$, $24n^2$		432	7(ii)(b), (c)
		11	$24n^2$, $48n^2$		$\bar{4}3m$	7(i)
		12*	$24n^2$		23	—
Trigonal	$\bar{6}m2$	17	$12n^2+72mn$		—	8(i)
Hexagonal	$6/mmm$	17	$72n^2+144mn$		—	—
			$216n^2+144mn$		—	—
Antiprisms						
Trigonal, octahedron	$m3m$	15	$24n^2$		$m3$	—
		16*	$8n^2$		422	—
		16	$24n^2$		432	5(iii)(b)
		17	16		$4/mmm$	5(iii)(a)
			$48n^2$, $144n^2$		—	5(iii)(c), (d)
Hexagonal	$\overline{12}m2$	16*	$72n^2$		622	—
		16	$216n^2$		622	—
		17	$144n^2$, $432n^2$		—	8(iv)
Pyramidal prisms						
Tetragonal	$4/mmm$	17	$48n^2+96mn$		—	8(ii)
			$144n^2+96mn$		—	—
Trigonal	$\bar{6}m2$	10*	$6n^2+12mn$		32	8(iii)
			$12n^2+24mn$		32	—
		11	$12n^2+24mn$		—	—
			$24n^2+24mn$	m odd	$3m$	—
				m even	—	—
		17	$36n^2+72mn$		—	—
			$108n^2+72mn$		—	—
Pentagonal	$\overline{10}m2$	17	$60n^2+120mn$		—	—
			$180n^2+120mn$		—	—
Pyramidal antiprisms						
Tetragonal 16-hedron	$\bar{8}m2$	16	$48n^2$		422	6(iii)(a)
		16*	$144n^2$		422	—
		17	$96n^2$, $288n^2$		—	6(iii)(b), (c)
Trigonal†	—	—	—		—	—
Pentagonal, icosahedron	$5m3m$	16	$60n^2$		532	4(i)
		16*	$180n^2$		532	—
		17	$120n^2$, $360n^2$		—	4(ii)

Table 1 (*cont.*).

Polyhedron type	Point group	Plane group	Number of units			Point group (if new)	Figure
Other polyhedra							
Tetrahedron	$\bar{4}3m$	16	$12n^2$			23	5(i)(*a*)
		16*	$36n^2$			23	—
		17	$24n^2$,	$72n^2$		—	5(i)(*b*), (*c*)
Trigonal dipyramid	$\bar{6}m2$	16*	$6n^2$			32	—
		16	$18n^2$			32	5(ii)(*b*)
		17	12,	$36n^2$,	$108n^2$	—	5(ii)(*a*), (*c*), (*d*)
Pentagonal dipyramid	$\overline{10}m2$	16*	$10n^2$			52	—
		16	$30n^2$			52	5(iv)(*b*)
		17	20,	$60n^2$,	$180n^2$	—	5(iv)(*a*), (*c*), (*d*)
12-hedron	$\bar{4}2m$	16	$36n^2$			2	6(i)(*a*)
		16*	$108n^2$			2	—
		17	$72n^2$,	$216n^2$		—	6(i)(*b*), (*c*)
14-hedron	$\bar{6}m2$	16	$42n^2$			32	6(ii)(*a*)
		16*	$126n^2$			32	—
		17	$84n^2$,	$252n^2$		—	6(ii)(*b*), (*c*)

l, m and n are positive integers. When m occurs in the number of units of a prism or pyramidal prism, it corresponds to the number of repeat units along its length

* Plane group mosaic as in Fig. 3.
† Formed from one octahedron and two tetrahedra. These compound polyhedra are not considered.

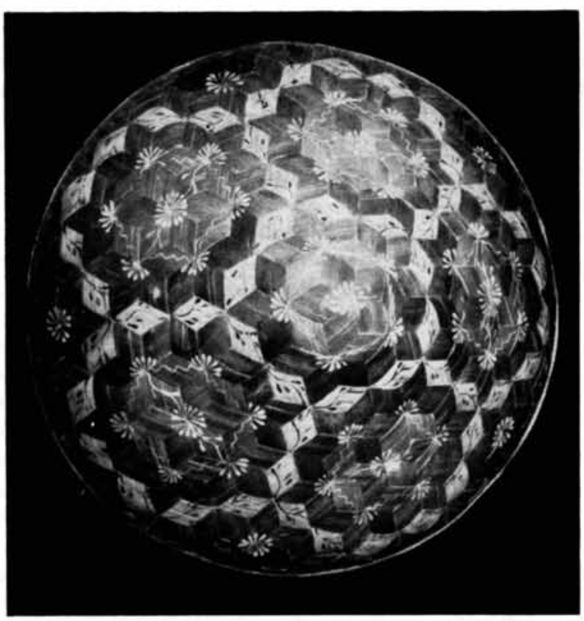

Fig. 9. A Chinese hat, viewed just off axis from above.

that some viruses have icosahedral 532 symmetry, e.g. bushy stunt virus (Caspar, 1956). Crick & Watson (1956) have suggested that the protein layer surrounding the ribonucleic acid centre of the virus is made up of a number of identical units packed in a symmetrical way. It is plausible that the nearest neighbour restriction here introduced may be applicable to these units of the protein shell. The shell would then be a 'plane group on a polyhedron'.

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